Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.4 Hyperbolic cotangent"

Test results for the 61 problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x\,\text{Coth}[a+b\,x]\,dx$$

Optimal (type 4, 45 leaves, 4 steps):

$$-\frac{x^2}{2}+\frac{x\,\text{Log}\left[1-e^{2\,(a+b\,x)}\right]}{b}+\frac{\text{PolyLog}\left[2,\,e^{2\,(a+b\,x)}\right]}{2\,b^2}$$

Result (type 4, 148 leaves):

$$\frac{1}{2\,b^2}\left(i\,b\,\pi\,x+b^2\,x^2\,\text{Coth}[a]-i\,\pi\,\text{Log}\left[1+e^{2\,b\,x}\right]+2\,b\,x\,\text{Log}\left[1-e^{-2\,(b\,x+\text{ArcTanh}[\text{Tanh}[a]])}\right]+\right.$$
$$i\,\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,\text{ArcTanh}[\text{Tanh}[a]]\,\left(b\,x+\text{Log}\left[1-e^{-2\,(b\,x+\text{ArcTanh}[\text{Tanh}[a]])}\right]-\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Tanh}[a]]]]\right)-$$
$$\left.\text{PolyLog}\left[2,\,e^{-2\,(b\,x+\text{ArcTanh}[\text{Tanh}[a]])}\right]-b^2\,e^{-\text{ArcTanh}[\text{Tanh}[a]]}\,x^2\,\text{Coth}[a]\,\sqrt{\text{Sech}[a]^2}\right)$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^2\,\text{Coth}[a+b\,x]^2\,dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\,\text{Coth}[a+b\,x]}{b}+\frac{2\,x\,\text{Log}\left[1-e^{2\,(a+b\,x)}\right]}{b^2}+\frac{\text{PolyLog}\left[2,\,e^{2\,(a+b\,x)}\right]}{b^3}$$

Result (type 4, 211 leaves):

$$\frac{x^3}{3}+\frac{x^2\,\text{Csch}[a]\,\text{Csch}[a+b\,x]\,\text{Sinh}[b\,x]}{b}+\Bigg(\text{Csch}[a]\,\text{Sech}[a]\,\Bigg(-b^2\,e^{-\text{ArcTanh}[\text{Tanh}[a]]}\,x^2+\frac{1}{\sqrt{1-\text{Tanh}[a]^2}}\,i\,\Big(-b\,x\,\big(-\pi+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\big)-\pi\,\text{Log}\big[1+e^{2\,b\,x}\big]-2\,\big(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]]\big)\,\text{Log}\big[1-e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]+\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\,\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Tanh}[a]]]]+i\,\text{PolyLog}\big[2,\,e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]\Big)\,\text{Tanh}[a]\Bigg)\Bigg)\Bigg/\Big(b^3\,\sqrt{\text{Sech}[a]^2\,\big(\text{Cosh}[a]^2-\text{Sinh}[a]^2\big)}\Big)$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x\,\text{Coth}[a+b\,x]^3\,dx$$

Optimal (type 4, 82 leaves, 7 steps):

$$\frac{x}{2\,b}-\frac{x^2}{2}-\frac{\text{Coth}[a+b\,x]}{2\,b^2}-\frac{x\,\text{Coth}[a+b\,x]^2}{2\,b}+\frac{x\,\text{Log}\big[1-e^{2\,(a+b\,x)}\big]}{b}+\frac{\text{PolyLog}\big[2,\,e^{2\,(a+b\,x)}\big]}{2\,b^2}$$

Result (type 4, 232 leaves):

$$\frac{1}{2}\,x^2\,\text{Coth}[a]-\frac{x\,\text{Csch}[a+b\,x]^2}{2\,b}+\frac{\text{Csch}[a]\,\text{Csch}[a+b\,x]\,\text{Sinh}[b\,x]}{2\,b^2}+\Bigg(\text{Csch}[a]\,\text{Sech}[a]\,\Bigg(-b^2\,e^{-\text{ArcTanh}[\text{Tanh}[a]]}\,x^2+\frac{1}{\sqrt{1-\text{Tanh}[a]^2}}\,i\,\Big(-b\,x\,\big(-\pi+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\big)-\pi\,\text{Log}\big[1+e^{2\,b\,x}\big]-2\,\big(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]]\big)\,\text{Log}\big[1-e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]+\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\,\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Tanh}[a]]]]+i\,\text{PolyLog}\big[2,\,e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]\Big)\,\text{Tanh}[a]\Bigg)\Bigg)\Bigg/\Big(2\,b^2\,\sqrt{\text{Sech}[a]^2\,\big(\text{Cosh}[a]^2-\text{Sinh}[a]^2\big)}\Big)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int\frac{(c+d\,x)^m}{a+a\,\text{Coth}[e+f\,x]}\,dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$\frac{(c+d\,x)^{1+m}}{2\,a\,d\,(1+m)}+\frac{2^{-2-m}\,e^{-2\,e+\frac{2\,c\,f}{d}}\,(c+d\,x)^m\,\left(\frac{f\,(c+d\,x)}{d}\right)^{-m}\,\text{Gamma}\left[1+m,\,\frac{2\,f\,(c+d\,x)}{d}\right]}{a\,f}$$

Result (type 4, 186 leaves):

$$\left(2^{-2-m} (c+d x)^m \left(-\frac{f (c+d x)}{d}\right)^m \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^{-m} \text{Csch}[e+f x] \left(d (1+m) \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right] \left(\text{Cosh}\left[e-\frac{c f}{d}\right]-\text{Sinh}\left[e-\frac{c f}{d}\right]\right)+2^{1+m} f \left(f \left(\frac{c}{d}+x\right)\right)^m (c+d x) \left(\text{Cosh}\left[e-\frac{c f}{d}\right]+\text{Sinh}\left[e-\frac{c f}{d}\right]\right)\right) \left(\text{Cosh}\left[f \left(\frac{c}{d}+x\right)\right]+\text{Sinh}\left[f \left(\frac{c}{d}+x\right)\right]\right)\right) \Big/ \left(a d f (1+m) (1+\text{Coth}[e+f x])\right)$$

Problem 35: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d x)^m}{(a+a \text{Coth}[e+f x])^2} \, dx$$

Optimal (type 4, 152 leaves, 4 steps):

$$\frac{(c+d x)^{1+m}}{4 a^2 d (1+m)}+\frac{2^{-2-m} e^{-2 e+\frac{2 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right]}{a^2 f}-\frac{4^{-2-m} e^{-4 e+\frac{4 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{4 f (c+d x)}{d}\right]}{a^2 f}$$

Result (type 1, 1 leaves):

???

Problem 36: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d x)^m}{(a+a \text{Coth}[e+f x])^3} \, dx$$

Optimal (type 4, 223 leaves, 5 steps):

$$\frac{(c+d x)^{1+m}}{8 a^3 d (1+m)}+\frac{3\times 2^{-4-m} e^{-2 e+\frac{2 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right]}{a^3 f}-\frac{3\times 2^{-5-2 m} e^{-4 e+\frac{4 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{4 f (c+d x)}{d}\right]}{a^3 f}+\frac{2^{-4-m}\times 3^{-1-m} e^{-6 e+\frac{6 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{6 f (c+d x)}{d}\right]}{a^3 f}$$

Result (type 1, 1 leaves):

???

Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+d x) (a+b \text{Coth}[e+f x]) \, dx$$

Optimal (type 4, 75 leaves, 6 steps):

$$\frac{a (c+d x)^2}{2 d}-\frac{b (c+d x)^2}{2 d}+\frac{b (c+d x) \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{b d \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}$$

Result (type 4, 227 leaves):

$$a c x+\frac{1}{2} a d x^2+\frac{1}{2} b d x^2 \text{Coth}[e]+\frac{b c \text{Log}[\text{Sinh}[e+f x]]}{f}+$$
$$\left(b d \text{Csch}[e] \text{Sech}[e] \left(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2+\frac{1}{\sqrt{1-\text{Tanh}[e]^2}} i \left(-f x \left(-\pi+2 i \text{ArcTanh}[\text{Tanh}[e]]\right)-\pi \text{Log}\left[1+e^{2 f x}\right]-\right.\right.\right.$$
$$2 \left(i f x+i \text{ArcTanh}[\text{Tanh}[e]]\right) \text{Log}\left[1-e^{2 i \left(i f x+i \text{ArcTanh}[\text{Tanh}[e]]\right)}\right]+\pi \text{Log}[\text{Cosh}[f x]]+2 i \text{ArcTanh}[\text{Tanh}[e]]$$
$$\left.\left.\left.\text{Log}[i \text{Sinh}[f x+\text{ArcTanh}[\text{Tanh}[e]]]]+i \text{PolyLog}\left[2, e^{2 i \left(i f x+i \text{ArcTanh}[\text{Tanh}[e]]\right)}\right]\right) \text{Tanh}[e]\right)\right) \Big/ \left(2 f^2 \sqrt{\text{Sech}[e]^2 \left(\text{Cosh}[e]^2-\text{Sinh}[e]^2\right)}\right)$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 (a+b \text{Coth}[e+f x])^2 \, dx$$

Optimal (type 4, 271 leaves, 15 steps):

$$-\frac{b^2 (c+d x)^3}{f}+\frac{a^2 (c+d x)^4}{4 d}-\frac{a b (c+d x)^4}{2 d}+\frac{b^2 (c+d x)^4}{4 d}-\frac{b^2 (c+d x)^3 \text{Coth}[e+f x]}{f}+\frac{3 b^2 d (c+d x)^2 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f^2}+$$
$$\frac{2 a b (c+d x)^3 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{3 b^2 d^2 (c+d x) \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^3}+\frac{3 a b d (c+d x)^2 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^2}-$$
$$\frac{3 b^2 d^3 \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^4}-\frac{3 a b d^2 (c+d x) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{f^3}+\frac{3 a b d^3 \text{PolyLog}\left[4, e^{2 (e+f x)}\right]}{2 f^4}$$

Result (type 4, 1084 leaves):

$$-\frac{1}{2\left(-1+e^{2e}\right)f}$$

$$b\, e^{2e}\Bigg(12 b c^2 d x+8 a c^3 f x+12 b c d^2 x^2+12 a c^2 d f x^2+4 b d^3 x^3+8 a c d^2 f x^3+2 a d^3 f x^4-4 a c^3 \text{Log}\left[1-e^{2(e+f x)}\right]+4 a c^3 e^{-2e} \text{Log}\left[1-e^{2(e+f x)}\right]-$$

$$\frac{6 b c^2 d \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{6 b c^2 d e^{-2e} \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-12 a c^2 d x \text{Log}\left[1-e^{2(e+f x)}\right]+12 a c^2 d e^{-2e} x \text{Log}\left[1-e^{2(e+f x)}\right]-$$

$$\frac{12 b c d^2 x \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{12 b c d^2 e^{-2e} x \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-12 a c d^2 x^2 \text{Log}\left[1-e^{2(e+f x)}\right]+$$

$$12 a c d^2 e^{-2e} x^2 \text{Log}\left[1-e^{2(e+f x)}\right]-\frac{6 b d^3 x^2 \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{6 b d^3 e^{-2e} x^2 \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-4 a d^3 x^3 \text{Log}\left[1-e^{2(e+f x)}\right]+$$

$$4 a d^3 e^{-2e} x^3 \text{Log}\left[1-e^{2(e+f x)}\right]-\frac{6 d e^{-2e}\left(-1+e^{2e}\right)(c+d x)\left(b d+a f (c+d x)\right) \text{PolyLog}\left[2, e^{2(e+f x)}\right]}{f^2}+$$

$$\frac{3 d^2 e^{-2e}\left(-1+e^{2e}\right)\left(b d+2 a f (c+d x)\right) \text{PolyLog}\left[3, e^{2(e+f x)}\right]}{f^3}-\frac{3 a d^3 \text{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^3}+\frac{3 a d^3 e^{-2e} \text{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^3}\Bigg)+$$

$$\frac{1}{8 f}\text{Csch}[e]\,\text{Csch}[e+f x]\Big(-4 a^2 c^3 f x \text{Cosh}[f x]-4 b^2 c^3 f x \text{Cosh}[f x]-6 a^2 c^2 d f x^2 \text{Cosh}[f x]-6 b^2 c^2 d f x^2 \text{Cosh}[f x]-$$

$$4 a^2 c d^2 f x^3 \text{Cosh}[f x]-4 b^2 c d^2 f x^3 \text{Cosh}[f x]-a^2 d^3 f x^4 \text{Cosh}[f x]-b^2 d^3 f x^4 \text{Cosh}[f x]+4 a^2 c^3 f x \text{Cosh}[2 e+f x]+$$

$$4 b^2 c^3 f x \text{Cosh}[2 e+f x]+6 a^2 c^2 d f x^2 \text{Cosh}[2 e+f x]+6 b^2 c^2 d f x^2 \text{Cosh}[2 e+f x]+4 a^2 c d^2 f x^3 \text{Cosh}[2 e+f x]+$$

$$4 b^2 c d^2 f x^3 \text{Cosh}[2 e+f x]+a^2 d^3 f x^4 \text{Cosh}[2 e+f x]+b^2 d^3 f x^4 \text{Cosh}[2 e+f x]+8 b^2 c^3 \text{Sinh}[f x]+24 b^2 c^2 d x \text{Sinh}[f x]+$$

$$8 a b c^3 f x \text{Sinh}[f x]+24 b^2 c d^2 x^2 \text{Sinh}[f x]+12 a b c^2 d f x^2 \text{Sinh}[f x]+8 b^2 d^3 x^3 \text{Sinh}[f x]+8 a b c d^2 f x^3 \text{Sinh}[f x]+$$

$$2 a b d^3 f x^4 \text{Sinh}[f x]+8 a b c^3 f x \text{Sinh}[2 e+f x]+12 a b c^2 d f x^2 \text{Sinh}[2 e+f x]+8 a b c d^2 f x^3 \text{Sinh}[2 e+f x]+2 a b d^3 f x^4 \text{Sinh}[2 e+f x]\Big)$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b \text{Coth}[e+f x]\right)^3 \, dx$$

Optimal (type 4, 556 leaves, 28 steps):

$$-\frac{3 b^3 d (c+d x)^2}{2 f^2}-\frac{3 a b^2 (c+d x)^3}{f}+\frac{b^3 (c+d x)^3}{2 f}+\frac{a^3 (c+d x)^4}{4 d}-\frac{3 a^2 b (c+d x)^4}{4 d}+\frac{3 a b^2 (c+d x)^4}{4 d}-\frac{b^3 (c+d x)^4}{4 d}-\frac{3 b^3 d (c+d x)^2 \text{Coth}[e+f x]}{2 f^2}-$$
$$\frac{3 a b^2 (c+d x)^3 \text{Coth}[e+f x]}{f}-\frac{b^3 (c+d x)^3 \text{Coth}[e+f x]^2}{2 f}+\frac{3 b^3 d^2 (c+d x) \text{Log}\left[1-e^{2 (e+f x)}\right]}{f^3}+\frac{9 a b^2 d (c+d x)^2 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f^2}+$$
$$\frac{3 a^2 b (c+d x)^3 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{b^3 (c+d x)^3 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{3 b^3 d^3 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^4}+\frac{9 a b^2 d^2 (c+d x) \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^3}+$$
$$\frac{9 a^2 b d (c+d x)^2 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}+\frac{3 b^3 d (c+d x)^2 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}-\frac{9 a b^2 d^3 \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^4}-$$
$$\frac{9 a^2 b d^2 (c+d x) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}-\frac{3 b^3 d^2 (c+d x) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}+\frac{9 a^2 b d^3 \text{PolyLog}\left[4, e^{2 (e+f x)}\right]}{4 f^4}+\frac{3 b^3 d^3 \text{PolyLog}\left[4, e^{2 (e+f x)}\right]}{4 f^4}$$

Result (type 4, 2043 leaves) :

$$\frac{\left(-b^3 c^3-3 b^3 c^2 d x-3 b^3 c d^2 x^2-b^3 d^3 x^3\right) \operatorname{Csch}[e+f x]^2}{2 f}-$$

$$\frac{1}{4\left(-1+e^{2 e}\right) f^2} b\, e^{2 e}\left(24 b^2 c d^2 x+72 a b c^2 d f x+24 a^2 c^3 f^2 x+8 b^2 c^3 f^2 x+12 b^2 d^3 x^2+72 a b c d^2 f x^2+36 a^2 c^2 d f^2 x^2+12 b^2 c^2 d f^2 x^2+\right.$$

$$24 a b d^3 f x^3+24 a^2 c d^2 f^2 x^3+8 b^2 c d^2 f^2 x^3+6 a^2 d^3 f^2 x^4+2 b^2 d^3 f^2 x^4-36 a b c^2 d \operatorname{Log}\left[1-e^{2(e+f x)}\right]+36 a b c^2 d\, e^{-2 e} \operatorname{Log}\left[1-e^{2(e+f x)}\right]-$$

$$\frac{12 b^2 c d^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{12 b^2 c d^2 e^{-2 e} \operatorname{Log}\left[1-e^{2(e+f x)}\right]}{f}-12 a^2 c^3 f \operatorname{Log}\left[1-e^{2(e+f x)}\right]-4 b^2 c^3 f \operatorname{Log}\left[1-e^{2(e+f x)}\right]+$$

$$12 a^2 c^3 e^{-2 e} f \operatorname{Log}\left[1-e^{2(e+f x)}\right]+4 b^2 c^3 e^{-2 e} f \operatorname{Log}\left[1-e^{2(e+f x)}\right]-72 a b c d^2 x \operatorname{Log}\left[1-e^{2(e+f x)}\right]+72 a b c d^2 e^{-2 e} x \operatorname{Log}\left[1-e^{2(e+f x)}\right]-$$

$$\frac{12 b^2 d^3 x \operatorname{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{12 b^2 d^3 e^{-2 e} x \operatorname{Log}\left[1-e^{2(e+f x)}\right]}{f}-36 a^2 c^2 d f x \operatorname{Log}\left[1-e^{2(e+f x)}\right]-12 b^2 c^2 d f x \operatorname{Log}\left[1-e^{2(e+f x)}\right]+$$

$$36 a^2 c^2 d e^{-2 e} f x \operatorname{Log}\left[1-e^{2(e+f x)}\right]+12 b^2 c^2 d e^{-2 e} f x \operatorname{Log}\left[1-e^{2(e+f x)}\right]-36 a b d^3 x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]+36 a b d^3 e^{-2 e} x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]-$$

$$36 a^2 c d^2 f x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]-12 b^2 c d^2 f x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]+36 a^2 c d^2 e^{-2 e} f x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]+12 b^2 c d^2 e^{-2 e} f x^2 \operatorname{Log}\left[1-e^{2(e+f x)}\right]-$$

$$12 a^2 d^3 f x^3 \operatorname{Log}\left[1-e^{2(e+f x)}\right]-4 b^2 d^3 f x^3 \operatorname{Log}\left[1-e^{2(e+f x)}\right]+12 a^2 d^3 e^{-2 e} f x^3 \operatorname{Log}\left[1-e^{2(e+f x)}\right]+4 b^2 d^3 e^{-2 e} f x^3 \operatorname{Log}\left[1-e^{2(e+f x)}\right]-$$

$$\frac{1}{f^2} 6 d\, e^{-2 e}\left(-1+e^{2 e}\right)\left(6 a b d f(c+d x)+3 a^2 f^2(c+d x)^2+b^2\left(d^2+c^2 f^2+2 c d f^2 x+d^2 f^2 x^2\right)\right) \operatorname{PolyLog}\left[2, e^{2(e+f x)}\right]+$$

$$\frac{6 d^2 e^{-2 e}\left(-1+e^{2 e}\right)\left(3 a b d+3 a^2 f(c+d x)+b^2 f(c+d x)\right) \operatorname{PolyLog}\left[3, e^{2(e+f x)}\right]}{f^2}-\frac{9 a^2 d^3 \operatorname{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^2}-$$

$$\left.\frac{3 b^2 d^3 \operatorname{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^2}+\frac{9 a^2 d^3 e^{-2 e} \operatorname{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^2}+\frac{3 b^2 d^3 e^{-2 e} \operatorname{PolyLog}\left[4, e^{2(e+f x)}\right]}{f^2}\right)+$$

$$\left(3 x^2\left(-a^3 c^2 d+3 a^2 b c^2 d-3 a b^2 c^2 d+b^3 c^2 d+a^3 c^2 d \operatorname{Cosh}[2 e]+3 a^2 b c^2 d \operatorname{Cosh}[2 e]+3 a b^2 c^2 d \operatorname{Cosh}[2 e]+b^3 c^2 d \operatorname{Cosh}[2 e]+\right.\right.$$

$$\left.\left.a^3 c^2 d \operatorname{Sinh}[2 e]+3 a^2 b c^2 d \operatorname{Sinh}[2 e]+3 a b^2 c^2 d \operatorname{Sinh}[2 e]+b^3 c^2 d \operatorname{Sinh}[2 e]\right)\right) /(2(-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e]))+$$

$$\left(x^3\left(-a^3 c d^2+3 a^2 b c d^2-3 a b^2 c d^2+b^3 c d^2+a^3 c d^2 \operatorname{Cosh}[2 e]+3 a^2 b c d^2 \operatorname{Cosh}[2 e]+3 a b^2 c d^2 \operatorname{Cosh}[2 e]+b^3 c d^2 \operatorname{Cosh}[2 e]+\right.\right.$$

$$\left.\left.a^3 c d^2 \operatorname{Sinh}[2 e]+3 a^2 b c d^2 \operatorname{Sinh}[2 e]+3 a b^2 c d^2 \operatorname{Sinh}[2 e]+b^3 c d^2 \operatorname{Sinh}[2 e]\right)\right) /(-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e])+$$

$$\left(x^4\left(-a^3 d^3+3 a^2 b d^3-3 a b^2 d^3+b^3 d^3+a^3 d^3 \operatorname{Cosh}[2 e]+3 a^2 b d^3 \operatorname{Cosh}[2 e]+3 a b^2 d^3 \operatorname{Cosh}[2 e]+b^3 d^3 \operatorname{Cosh}[2 e]+\right.\right.$$

$$\left.\left.a^3 d^3 \operatorname{Sinh}[2 e]+3 a^2 b d^3 \operatorname{Sinh}[2 e]+3 a b^2 d^3 \operatorname{Sinh}[2 e]+b^3 d^3 \operatorname{Sinh}[2 e]\right)\right) /(4(-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e]))+$$

$$x\left(a^3 c^3+3 a b^2 c^3+\frac{3 a^2 b c^3}{-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e]}+\frac{3 a^2 b c^3 \operatorname{Cosh}[2 e]+3 a^2 b c^3 \operatorname{Sinh}[2 e]}{-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e]}+\right.$$

$$\frac{2 b^3 c^3 \operatorname{Cosh}[2 e]+2 b^3 c^3 \operatorname{Sinh}[2 e]}{(-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e])(1+\operatorname{Cosh}[2 e]+\operatorname{Cosh}[4 e]+\operatorname{Sinh}[2 e]+\operatorname{Sinh}[4 e])}+$$

$$\frac{2 b^3 c^3 \operatorname{Cosh}[4 e]+2 b^3 c^3 \operatorname{Sinh}[4 e]}{(-1+\operatorname{Cosh}[2 e]+\operatorname{Sinh}[2 e])(1+\operatorname{Cosh}[2 e]+\operatorname{Cosh}[4 e]+\operatorname{Sinh}[2 e]+\operatorname{Sinh}[4 e])}+$$

$$\left.\frac{b^3 c^3}{-1+\operatorname{Cosh}[6 e]+\operatorname{Sinh}[6 e]}+\frac{b^3 c^3 \operatorname{Cosh}[6 e]+b^3 c^3 \operatorname{Sinh}[6 e]}{-1+\operatorname{Cosh}[6 e]+\operatorname{Sinh}[6 e]}\right)+\frac{1}{2 f^2}$$

$$3 \operatorname{Csch}[e] \operatorname{Csch}[e+f x]\left(b^3 c^2 d \operatorname{Sinh}[f x]+2 a b^2 c^3 f \operatorname{Sinh}[f x]+2 b^3 c d^2 x \operatorname{Sinh}[f x]+6 a b^2 c^2 d f x \operatorname{Sinh}[f x]+\right.$$

$$\left.b^3 d^3 x^2 \operatorname{Sinh}[f x]+6 a b^2 c d^2 f x^2 \operatorname{Sinh}[f x]+2 a b^2 d^3 f x^3 \operatorname{Sinh}[f x]\right)$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+d x)^2 (a+b \operatorname{Coth}[e+f x])^3 \, dx$$

Optimal (type 4, 401 leaves, 22 steps):

$$\frac{b^3 c d x}{f}+\frac{b^3 d^2 x^2}{2 f}-\frac{3 a b^2 (c+d x)^2}{f}+\frac{a^3 (c+d x)^3}{3 d}-\frac{a^2 b (c+d x)^3}{d}+\frac{a b^2 (c+d x)^3}{d}-\frac{b^3 (c+d x)^3}{3 d}-\frac{b^3 d (c+d x) \operatorname{Coth}[e+f x]}{f^2}-\frac{3 a b^2 (c+d x)^2 \operatorname{Coth}[e+f x]}{f}-\frac{b^3 (c+d x)^2 \operatorname{Coth}[e+f x]^2}{2 f}+\frac{6 a b^2 d (c+d x) \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f^2}+\frac{3 a^2 b (c+d x)^2 \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{b^3 (c+d x)^2 \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{b^3 d^2 \operatorname{Log}[\operatorname{Sinh}[e+f x]]}{f^3}+\frac{3 a b^2 d^2 \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^3}+\frac{3 a^2 b d (c+d x) \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^2}+\frac{b^3 d (c+d x) \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^2}-\frac{3 a^2 b d^2 \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}-\frac{b^3 d^2 \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}$$

Result (type 4, 1887 leaves):

$$-\frac{1}{4 f^3} a^2 b d^2 e^{-e} \operatorname{Csch}[e] \left(2 f^2 x^2 \left(2 e^{2 e} f x-3 \left(-1+e^{2 e}\right) \operatorname{Log}\left[1-e^{2 (e+f x)}\right]\right)-6 \left(-1+e^{2 e}\right) f x \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]+3 \left(-1+e^{2 e}\right) \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]\right)-\frac{1}{12 f^3} b^3 d^2 e^{-e} \operatorname{Csch}[e] \left(2 f^2 x^2 \left(2 e^{2 e} f x-3 \left(-1+e^{2 e}\right) \operatorname{Log}\left[1-e^{2 (e+f x)}\right]\right)-6 \left(-1+e^{2 e}\right) f x \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]+3 \left(-1+e^{2 e}\right) \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]\right)-\frac{b^3 d^2 \operatorname{Csch}[e] (-f x \operatorname{Cosh}[e]+\operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e]+\operatorname{Cosh}[e] \operatorname{Sinh}[f x]] \operatorname{Sinh}[e])}{f^3 \left(-\operatorname{Cosh}[e]^2+\operatorname{Sinh}[e]^2\right)}-\frac{6 a b^2 c d \operatorname{Csch}[e] (-f x \operatorname{Cosh}[e]+\operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e]+\operatorname{Cosh}[e] \operatorname{Sinh}[f x]] \operatorname{Sinh}[e])}{f^2 \left(-\operatorname{Cosh}[e]^2+\operatorname{Sinh}[e]^2\right)}-\frac{3 a^2 b c^2 \operatorname{Csch}[e] (-f x \operatorname{Cosh}[e]+\operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e]+\operatorname{Cosh}[e] \operatorname{Sinh}[f x]] \operatorname{Sinh}[e])}{f \left(-\operatorname{Cosh}[e]^2+\operatorname{Sinh}[e]^2\right)}-\frac{b^3 c^2 \operatorname{Csch}[e] (-f x \operatorname{Cosh}[e]+\operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e]+\operatorname{Cosh}[e] \operatorname{Sinh}[f x]] \operatorname{Sinh}[e])}{f \left(-\operatorname{Cosh}[e]^2+\operatorname{Sinh}[e]^2\right)}+$$

$$\frac{1}{12 f^2} \operatorname{Csch}[e] \operatorname{Csch}[e+f x]^2 \left(-6 b^3 c d \operatorname{Cosh}[e]-18 a b^2 c^2 f \operatorname{Cosh}[e]-6 b^3 d^2 x \operatorname{Cosh}[e]-36 a b^2 c d f x \operatorname{Cosh}[e]-18 a^2 b c^2 f^2 x \operatorname{Cosh}[e]-\right.$$
$$6 b^3 c^2 f^2 x \operatorname{Cosh}[e]-18 a b^2 d^2 f x^2 \operatorname{Cosh}[e]-18 a^2 b c d f^2 x^2 \operatorname{Cosh}[e]-6 b^3 c d f^2 x^2 \operatorname{Cosh}[e]-6 a^2 b d^2 f^2 x^3 \operatorname{Cosh}[e]-2 b^3 d^2 f^2 x^3 \operatorname{Cosh}[e]+$$
$$6 b^3 c d \operatorname{Cosh}[e+2 f x]+18 a b^2 c^2 f \operatorname{Cosh}[e+2 f x]+6 b^3 d^2 x \operatorname{Cosh}[e+2 f x]+36 a b^2 c d f x \operatorname{Cosh}[e+2 f x]+9 a^2 b c^2 f^2 x \operatorname{Cosh}[e+2 f x]+$$
$$3 b^3 c^2 f^2 x \operatorname{Cosh}[e+2 f x]+18 a b^2 d^2 f x^2 \operatorname{Cosh}[e+2 f x]+9 a^2 b c d f^2 x^2 \operatorname{Cosh}[e+2 f x]+3 b^3 c d f^2 x^2 \operatorname{Cosh}[e+2 f x]+3 a^2 b d^2 f^2 x^3$$
$$\operatorname{Cosh}[e+2 f x]+b^3 d^2 f^2 x^3 \operatorname{Cosh}[e+2 f x]+9 a^2 b c^2 f^2 x \operatorname{Cosh}[3 e+2 f x]+3 b^3 c^2 f^2 x \operatorname{Cosh}[3 e+2 f x]+9 a^2 b c d f^2 x^2 \operatorname{Cosh}[3 e+2 f x]+$$
$$3 b^3 c d f^2 x^2 \operatorname{Cosh}[3 e+2 f x]+3 a^2 b d^2 f^2 x^3 \operatorname{Cosh}[3 e+2 f x]+b^3 d^2 f^2 x^3 \operatorname{Cosh}[3 e+2 f x]-6 b^3 c^2 f \operatorname{Sinh}[e]-12 b^3 c d f x \operatorname{Sinh}[e]-$$
$$6 a^3 c^2 f^2 x \operatorname{Sinh}[e]-18 a b^2 c^2 f^2 x \operatorname{Sinh}[e]-6 b^3 d^2 f x^2 \operatorname{Sinh}[e]-6 a^3 c d f^2 x^2 \operatorname{Sinh}[e]-18 a b^2 c d f^2 x^2 \operatorname{Sinh}[e]-2 a^3 d^2 f^2 x^3 \operatorname{Sinh}[e]-$$

$6 a b^2 d^2 f^2 x^3 \text{Sinh}[e] - 3 a^3 c^2 f^2 x \text{Sinh}[e + 2 f x] - 9 a b^2 c^2 f^2 x \text{Sinh}[e + 2 f x] - 3 a^3 c d f^2 x^2 \text{Sinh}[e + 2 f x] - 9 a b^2 c d f^2 x^2 \text{Sinh}[e + 2 f x] - a^3 d^2 f^2 x^3 \text{Sinh}[e + 2 f x] - 3 a b^2 d^2 f^2 x^3 \text{Sinh}[e + 2 f x] + 3 a^3 c^2 f^2 x \text{Sinh}[3 e + 2 f x] + 9 a b^2 c^2 f^2 x \text{Sinh}[3 e + 2 f x] + 3 a^3 c d f^2 x^2 \text{Sinh}[3 e + 2 f x] + 9 a b^2 c d f^2 x^2 \text{Sinh}[3 e + 2 f x] + a^3 d^2 f^2 x^3 \text{Sinh}[3 e + 2 f x] + 3 a b^2 d^2 f^2 x^3 \text{Sinh}[3 e + 2 f x]\Big) +$

$\Big(3 a b^2 d^2 \text{Csch}[e] \text{Sech}[e] \Big(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \text{Tanh}[e]^2}} i \big(-f x (-\pi + 2 i \text{ArcTanh}[\text{Tanh}[e]]) - \pi \text{Log}[1 + e^{2 f x}] - 2 (i f x + i \text{ArcTanh}[\text{Tanh}[e]]) \text{Log}[1 - e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}] + \pi \text{Log}[\text{Cosh}[f x]] + 2 i \text{ArcTanh}[\text{Tanh}[e]] \text{Log}[i \text{Sinh}[f x + \text{ArcTanh}[\text{Tanh}[e]]]] + i \text{PolyLog}[2, e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}]\big) \text{Tanh}[e]\Big)\Big) \Big/$

$\left(f^3 \sqrt{\text{Sech}[e]^2 (\text{Cosh}[e]^2 - \text{Sinh}[e]^2)}\right) + \Big(3 a^2 b c d \text{Csch}[e] \text{Sech}[e] \Big(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \text{Tanh}[e]^2}} i \big(-f x (-\pi + 2 i \text{ArcTanh}[\text{Tanh}[e]]) - \pi \text{Log}[1 + e^{2 f x}] - 2 (i f x + i \text{ArcTanh}[\text{Tanh}[e]]) \text{Log}[1 - e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}] + \pi \text{Log}[\text{Cosh}[f x]] + 2 i \text{ArcTanh}[\text{Tanh}[e]] \text{Log}[i \text{Sinh}[f x + \text{ArcTanh}[\text{Tanh}[e]]]] + i \text{PolyLog}[2, e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}]\big) \text{Tanh}[e]\Big)\Big) \Big/$

$\left(f^2 \sqrt{\text{Sech}[e]^2 (\text{Cosh}[e]^2 - \text{Sinh}[e]^2)}\right) + \Big(b^3 c d \text{Csch}[e] \text{Sech}[e] \Big(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \text{Tanh}[e]^2}} i \big(-f x (-\pi + 2 i \text{ArcTanh}[\text{Tanh}[e]]) - \pi \text{Log}[1 + e^{2 f x}] - 2 (i f x + i \text{ArcTanh}[\text{Tanh}[e]]) \text{Log}[1 - e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}] + \pi \text{Log}[\text{Cosh}[f x]] + 2 i \text{ArcTanh}[\text{Tanh}[e]] \text{Log}[i \text{Sinh}[f x + \text{ArcTanh}[\text{Tanh}[e]]]] + i \text{PolyLog}[2, e^{2 i (i f x + i \text{ArcTanh}[\text{Tanh}[e]])}]\big) \text{Tanh}[e]\Big)\Big) \Big/ \left(f^2 \sqrt{\text{Sech}[e]^2 (\text{Cosh}[e]^2 - \text{Sinh}[e]^2)}\right)$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + b \text{Coth}[e + f x])^2} \, dx$$

Optimal (type 4, 638 leaves, 28 steps):

$$-\frac{2 b^2 (c+d x)^3}{(a^2-b^2)^2 f}+\frac{2 b^2 (c+d x)^3}{(a-b) (a+b)^2 \left(a-b-(a+b) e^{2 e+2 f x}\right) f}+\frac{(c+d x)^4}{4 (a-b)^2 d}+\frac{3 b^2 d (c+d x)^2 \operatorname{Log}\left[1-\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a^2-b^2)^2 f^2}-\frac{2 b (c+d x)^3 \operatorname{Log}\left[1-\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f}+$$

$$\frac{2 b^2 (c+d x)^3 \operatorname{Log}\left[1-\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a^2-b^2)^2 f}+\frac{3 b^2 d^2 (c+d x) \operatorname{PolyLog}\left[2,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a^2-b^2)^2 f^3}-\frac{3 b d (c+d x)^2 \operatorname{PolyLog}\left[2,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f^2}+$$

$$\frac{3 b^2 d (c+d x)^2 \operatorname{PolyLog}\left[2,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a^2-b^2)^2 f^2}-\frac{3 b^2 d^3 \operatorname{PolyLog}\left[3,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{2 (a^2-b^2)^2 f^4}+\frac{3 b d^2 (c+d x) \operatorname{PolyLog}\left[3,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f^3}-$$

$$\frac{3 b^2 d^2 (c+d x) \operatorname{PolyLog}\left[3,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{(a^2-b^2)^2 f^3}-\frac{3 b d^3 \operatorname{PolyLog}\left[4,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{2 (a-b)^2 (a+b) f^4}+\frac{3 b^2 d^3 \operatorname{PolyLog}\left[4,\frac{(a+b) e^{2 e+2 f x}}{a-b}\right]}{2 (a^2-b^2)^2 f^4}$$

Result (type 4, 2115 leaves) :

$$-\frac{1}{2\,(a-b)^2\,(a+b)^2\,\left(a\,(-1+e^{2e})+b\,(1+e^{2e})\right)\,f^4}$$

$$b\,\Bigg(12\,a\,b\,c^2\,d\,e^{2e}\,f^3\,x+12\,b^2\,c^2\,d\,e^{2e}\,f^3\,x-8\,a^2\,c^3\,e^{2e}\,f^4\,x-8\,a\,b\,c^3\,e^{2e}\,f^4\,x+12\,a\,b\,c\,d^2\,e^{2e}\,f^3\,x^2+12\,b^2\,c\,d^2\,e^{2e}\,f^3\,x^2-$$

$$12\,a^2\,c^2\,d\,e^{2e}\,f^4\,x^2-12\,a\,b\,c^2\,d\,e^{2e}\,f^4\,x^2+4\,a\,b\,d^3\,e^{2e}\,f^3\,x^3+4\,b^2\,d^3\,e^{2e}\,f^3\,x^3-8\,a^2\,c\,d^2\,e^{2e}\,f^4\,x^3-8\,a\,b\,c\,d^2\,e^{2e}\,f^4\,x^3-$$

$$2\,a^2\,d^3\,e^{2e}\,f^4\,x^4-2\,a\,b\,d^3\,e^{2e}\,f^4\,x^4+12\,a\,b\,c\,d^2\,f^2\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-12\,b^2\,c\,d^2\,f^2\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-$$

$$12\,a\,b\,c\,d^2\,e^{2e}\,f^2\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-12\,b^2\,c\,d^2\,e^{2e}\,f^2\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-12\,a^2\,c^2\,d\,f^3\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+$$

$$12\,a\,b\,c^2\,d\,f^3\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+12\,a^2\,c^2\,d\,e^{2e}\,f^3\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+12\,a\,b\,c^2\,d\,e^{2e}\,f^3\,x\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+$$

$$6\,a\,b\,d^3\,f^2\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-6\,b^2\,d^3\,f^2\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-6\,a\,b\,d^3\,e^{2e}\,f^2\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-$$

$$6\,b^2\,d^3\,e^{2e}\,f^2\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-12\,a^2\,c\,d^2\,f^3\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+12\,a\,b\,c\,d^2\,f^3\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+$$

$$12\,a^2\,c\,d^2\,e^{2e}\,f^3\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+12\,a\,b\,c\,d^2\,e^{2e}\,f^3\,x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]-4\,a^2\,d^3\,f^3\,x^3\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+$$

$$4\,a\,b\,d^3\,f^3\,x^3\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+4\,a^2\,d^3\,e^{2e}\,f^3\,x^3\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+4\,a\,b\,d^3\,e^{2e}\,f^3\,x^3\,\text{Log}\left[1+\frac{(a+b)\,e^{2(e+f\,x)}}{-a+b}\right]+$$

$$6\,a\,b\,c^2\,d\,f^2\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]-6\,b^2\,c^2\,d\,f^2\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]-$$

$$6\,a\,b\,c^2\,d\,e^{2e}\,f^2\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]-6\,b^2\,c^2\,d\,e^{2e}\,f^2\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]-$$

$$4\,a^2\,c^3\,f^3\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]+4\,a\,b\,c^3\,f^3\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]+$$

$$4\,a^2\,c^3\,e^{2e}\,f^3\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]+4\,a\,b\,c^3\,e^{2e}\,f^3\,\text{Log}\left[a\,(-1+e^{2(e+f\,x)})+b\,(1+e^{2(e+f\,x)})\right]+$$

$$6\,d\,\left(a\,(-1+e^{2e})+b\,(1+e^{2e})\right)\,f\,(c+d\,x)\,\left(-b\,d+a\,f\,(c+d\,x)\right)\,\text{PolyLog}\left[2,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]-$$

$$3\,d^2\,\left(a\,(-1+e^{2e})+b\,(1+e^{2e})\right)\,\left(-b\,d+2\,a\,f\,(c+d\,x)\right)\,\text{PolyLog}\left[3,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]-3\,a^2\,d^3\,\text{PolyLog}\left[4,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]+$$

$$3\,a\,b\,d^3\,\text{PolyLog}\left[4,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]+3\,a^2\,d^3\,e^{2e}\,\text{PolyLog}\left[4,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]+3\,a\,b\,d^3\,e^{2e}\,\text{PolyLog}\left[4,\frac{(a+b)\,e^{2(e+f\,x)}}{a-b}\right]\Bigg)+$$

$$\big(-4\,a^2\,c^3\,f\,x\,\text{Cosh}[f\,x]-4\,b^2\,c^3\,f\,x\,\text{Cosh}[f\,x]-6\,a^2\,c^2\,d\,f\,x^2\,\text{Cosh}[f\,x]-6\,b^2\,c^2\,d\,f\,x^2\,\text{Cosh}[f\,x]-4\,a^2\,c\,d^2\,f\,x^3\,\text{Cosh}[f\,x]-$$

$$4\,b^2\,c\,d^2\,f\,x^3\,\text{Cosh}[f\,x]-a^2\,d^3\,f\,x^4\,\text{Cosh}[f\,x]-b^2\,d^3\,f\,x^4\,\text{Cosh}[f\,x]+4\,a^2\,c^3\,f\,x\,\text{Cosh}[2\,e+f\,x]-$$

$$4\,b^2\,c^3\,f\,x\,\text{Cosh}[2\,e+f\,x]+6\,a^2\,c^2\,d\,f\,x^2\,\text{Cosh}[2\,e+f\,x]-6\,b^2\,c^2\,d\,f\,x^2\,\text{Cosh}[2\,e+f\,x]+$$

$$4\,a^2\,c\,d^2\,f\,x^3\,\text{Cosh}[2\,e+f\,x]-4\,b^2\,c\,d^2\,f\,x^3\,\text{Cosh}[2\,e+f\,x]+a^2\,d^3\,f\,x^4\,\text{Cosh}[2\,e+f\,x]-b^2\,d^3\,f\,x^4\,\text{Cosh}[2\,e+f\,x]+$$

$$8\,b^2\,c^3\,\text{Sinh}[f\,x]+24\,b^2\,c^2\,d\,x\,\text{Sinh}[f\,x]-8\,a\,b\,c^3\,f\,x\,\text{Sinh}[f\,x]+24\,b^2\,c\,d^2\,x^2\,\text{Sinh}[f\,x]-$$

$$12\,a\,b\,c^2\,d\,f\,x^2\,\text{Sinh}[f\,x]+8\,b^2\,d^3\,x^3\,\text{Sinh}[f\,x]-8\,a\,b\,c\,d^2\,f\,x^3\,\text{Sinh}[f\,x]-2\,a\,b\,d^3\,f\,x^4\,\text{Sinh}[f\,x]\big)\,\big/$$

$$\left(8\,(a-b)\,(a+b)\,f\,(b\,\text{Cosh}[e]+a\,\text{Sinh}[e])\,(b\,\text{Cosh}[e+f\,x]+a\,\text{Sinh}[e+f\,x])\right)$$

Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{c+d\,x}{\left(a+b\,\text{Coth}[e+f\,x]\right)^2}\,dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$-\frac{(c+d\,x)^2}{2\,(a^2-b^2)\,d}+\frac{(b\,d-2\,a\,c\,f-2\,a\,d\,f\,x)^2}{4\,a\,(a-b)\,(a+b)^2\,d\,f^2}+\frac{b\,(c+d\,x)}{(a^2-b^2)\,f\,(a+b\,\text{Coth}[e+f\,x])}+$$
$$\frac{b\,(b\,d-2\,a\,c\,f-2\,a\,d\,f\,x)\,\text{Log}\left[1-\frac{(a-b)\,e^{-2\,(e+f\,x)}}{a+b}\right]}{(a^2-b^2)^2\,f^2}+\frac{a\,b\,d\,\text{PolyLog}\left[2,\frac{(a-b)\,e^{-2\,(e+f\,x)}}{a+b}\right]}{(a^2-b^2)^2\,f^2}$$

Result (type 4, 737 leaves):

$$\frac{(e+f x)\ (-2 d e+2 c f+d\ (e+f x))\ \text{Csch}[e+f x]^2\ (b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])^2}{2\ (-a+b)\ (a+b)\ f^2\ (a+b \text{Coth}[e+f x])^2}+$$

$$\left(b d \text{Csch}[e+f x]^2\ (-a\ (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]])\ (b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])^2\right) \Big/ \left((-a+b)\ (a+b)\ (-a^2+b^2)\ f^2\ (a+b \text{Coth}[e+f x])^2\right)+$$

$$\left(2 a d e \text{Csch}[e+f x]^2\ (-a\ (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]])\ (b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])^2\right) \Big/ \left((-a+b)\ (a+b)\ (-a^2+b^2)\ f^2\ (a+b \text{Coth}[e+f x])^2\right)-$$

$$\left(2 a c \text{Csch}[e+f x]^2\ (-a\ (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]])\ (b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])^2\right) \Big/ \left((-a+b)\ (a+b)\ (-a^2+b^2)\ f\ (a+b \text{Coth}[e+f x])^2\right)+$$

$$\left(d \text{Csch}[e+f x]^2 \left(-e^{-\text{ArcTanh}\left[\frac{b}{a}\right]}\ (e+f x)^2+\frac{1}{a \sqrt{1-\frac{b^2}{a^2}}}\right.\right.$$

$$i\, b \left(-(e+f x)\ \left(-\pi+2 i \text{ArcTanh}\left[\frac{b}{a}\right]\right)-\pi \text{Log}\left[1+e^{2 (e+f x)}\right]-2 \left(i\ (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right) \text{Log}\left[1-e^{2 i \left(i\ (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right)}\right]+\right.$$

$$\left.\left.\pi \text{Log}[\text{Cosh}[e+f x]]+2 i \text{ArcTanh}\left[\frac{b}{a}\right] \text{Log}\left[i \text{Sinh}\left[e+f x+\text{ArcTanh}\left[\frac{b}{a}\right]\right]\right]+i \text{PolyLog}\left[2, e^{2 i \left(i\ (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right)}\right]\right)\right)$$

$$\left.(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])^2\right) \Big/ \left((-a+b)\ (a+b) \sqrt{\frac{a^2-b^2}{a^2}}\ f^2\ (a+b \text{Coth}[e+f x])^2\right)+$$

$$\left(\text{Csch}[e+f x]^2\ (b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x])\ (b d e \text{Sinh}[e+f x]-b c f \text{Sinh}[e+f x]-b d\ (e+f x) \text{Sinh}[e+f x])\right) \Big/ \left((-a+b)\ (a+b)\ f^2\ (a+b \text{Coth}[e+f x])^2\right)$$

Problem 60: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c+d x)\ (a+b \text{Coth}[e+f x])^2}\, dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c+d x)\ (a+b \text{Coth}[e+f x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 61: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c+d\,x)^2\,(a+b\,\text{Coth}[e+f\,x])^2}\,dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c+d\,x)^2\,(a+b\,\text{Coth}[e+f\,x])^2},x\right]$$

Result (type 1, 1 leaves):

???

Test results for the 224 problems in "6.4.2 Hyperbolic cotangent functions.m"

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int (1+\text{Coth}[x])^{7/2}\,dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$8\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-8\sqrt{1+\text{Coth}[x]}-\frac{4}{3}(1+\text{Coth}[x])^{3/2}-\frac{2}{5}(1+\text{Coth}[x])^{5/2}$$

Result (type 3, 101 leaves):

$$-\left(\left(2\,(1+\text{Coth}[x])^{7/2}\left(4\left((-15+15\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]+19\sqrt{i\,(1+\text{Coth}[x])}\right)\text{Sinh}[x]^3+\sqrt{i\,(1+\text{Coth}[x])}\,\text{Sinh}[x]\,(3+8\,\text{Sinh}[2\,x])\right)\right)\Big/\left(15\sqrt{i\,(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])^3\right)\right)$$

Problem 71: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (1+\text{Coth}[x])^{5/2}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$4\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-4\sqrt{1+\text{Coth}[x]}-\frac{2}{3}(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 92 leaves):

$$-\left(\left(2(1+\text{Coth}[x])^{5/2}\,\text{Sinh}[x]\left(\text{Cosh}[x]\sqrt{i(1+\text{Coth}[x])}+\left((-6+6i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\text{Coth}[x])}\right]+7\sqrt{i(1+\text{Coth}[x])}\right)\text{Sinh}[x]\right)\right)\Big/\left(3\sqrt{i(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])^2\right)\right)$$

Problem 72: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int(1+\text{Coth}[x])^{3/2}\,dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 69 leaves):

$$-\frac{2(1+\text{Coth}[x])^{3/2}\left((-1+i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\text{Coth}[x])}\right]+\sqrt{i(1+\text{Coth}[x])}\right)\text{Sinh}[x]}{\sqrt{i(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])}$$

Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]$$

Result (type 3, 45 leaves):

$$\frac{(1+i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\text{Coth}[x])}\right](1+\text{Coth}[x])^{3/2}}{(i(1+\text{Coth}[x]))^{3/2}}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+\text{Coth}[x]}} \, dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{1}{\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 51 leaves):

$$\frac{-2-(1+i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\sqrt{i\,(1+\text{Coth}[x])}}{2\sqrt{1+\text{Coth}[x]}}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\text{Coth}[x])^{3/2}} \, dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\sqrt{2}} - \frac{1}{3\,(1+\text{Coth}[x])^{3/2}} - \frac{1}{2\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 86 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}} + \left(\frac{1}{6}-\frac{i}{6}\right)\left(-4+5\,\text{Cosh}[2x]-\text{Cosh}[4x]-5\,\text{Sinh}[2x]+\text{Sinh}[4x]\right)\right)$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\text{Coth}[x])^{5/2}} \, dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{4\sqrt{2}}-\frac{1}{5\,(1+\text{Coth}[x])^{5/2}}-\frac{1}{6\,(1+\text{Coth}[x])^{3/2}}-\frac{1}{4\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 94 leaves):

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\,(1+\text{Coth}[x])^{3/2}}{(i\,(1+\text{Coth}[x]))^{3/2}}-$$
$$\frac{1}{60}\sqrt{1+\text{Coth}[x]}\,(\text{Cosh}[3x]-\text{Sinh}[3x])\,(-10\,\text{Cosh}[x]+10\,\text{Cosh}[3x]-24\,\text{Sinh}[x]+13\,\text{Sinh}[3x])$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int(a+b\,\text{Coth}[c+d\,x])^5\,dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$a\,(a^4+10\,a^2\,b^2+5\,b^4)\,x-\frac{4\,a\,b^2\,(a^2+b^2)\,\text{Coth}[c+d\,x]}{d}-\frac{b\,(3\,a^2+b^2)\,(a+b\,\text{Coth}[c+d\,x])^2}{2\,d}-$$
$$\frac{2\,a\,b\,(a+b\,\text{Coth}[c+d\,x])^3}{3\,d}-\frac{b\,(a+b\,\text{Coth}[c+d\,x])^4}{4\,d}+\frac{b\,(5\,a^4+10\,a^2\,b^2+b^4)\,\text{Log}[\text{Sinh}[c+d\,x]]}{d}$$

Result (type 3, 367 leaves):

$$-\frac{b^5\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]}{4\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-\frac{5\,a\,b^4\,\text{Cosh}[c+d\,x]\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^2}{3\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-$$
$$\frac{b^3\,(5\,a^2+b^2)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^3}{d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-\frac{10\,(3\,a^3\,b^2\,\text{Cosh}[c+d\,x]+2\,a\,b^4\,\text{Cosh}[c+d\,x])\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^4}{3\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}+$$
$$\frac{a\,(a^4+10\,a^2\,b^2+5\,b^4)\,(c+d\,x)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^5}{d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}+\frac{(5\,a^4\,b+10\,a^2\,b^3+b^5)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Log}[\text{Sinh}[c+d\,x]]\,\text{Sinh}[c+d\,x]^5}{d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,\text{Coth}[c+d\,x])^4}\,dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(a^4+6 a^2 b^2+b^4\right) x}{\left(a^2-b^2\right)^4}+\frac{b}{3 \left(a^2-b^2\right) d \left(a+b \operatorname{Coth}[c+d x]\right)^3}+\frac{a b}{\left(a^2-b^2\right)^2 d \left(a+b \operatorname{Coth}[c+d x]\right)^2}+\frac{b \left(3 a^2+b^2\right)}{\left(a^2-b^2\right)^3 d \left(a+b \operatorname{Coth}[c+d x]\right)}-\frac{4 a b \left(a^2+b^2\right) \operatorname{Log}[b \operatorname{Cosh}[c+d x]+a \operatorname{Sinh}[c+d x]]}{\left(a^2-b^2\right)^4 d}$$

Result (type 3, 440 leaves):

$$\frac{1}{3 (a-b)^4 (a+b)^4 d \left(a+b \operatorname{Coth}[c+d x]\right)^3}\Big(b^3 \left(6 a^4-7 a^2 b^2+b^4\right) \operatorname{Csch}[c+d x]^2+ 3 b^3 \operatorname{Coth}[c+d x]^3 \left(\left(a^4+6 a^2 b^2+b^4\right) (c+d x)-4 a b \left(a^2+b^2\right) \operatorname{Log}[b \operatorname{Cosh}[c+d x]+a \operatorname{Sinh}[c+d x]]\right)+b^2 \operatorname{Coth}[c+d x]^2 \left(18 a^4 b-14 a^2 b^3-4 b^5+9 a^5 (c+d x)+54 a^3 b^2 (c+d x)+9 a b^4 (c+d x)-36 a^2 b \left(a^2+b^2\right) \operatorname{Log}[b \operatorname{Cosh}[c+d x]+a \operatorname{Sinh}[c+d x]]\right)+ a b \operatorname{Coth}[c+d x] \left(36 a^4 b-28 a^2 b^3-8 b^5+9 a^5 c+54 a^3 b^2 c+9 a b^4 c+9 a^5 d x+54 a^3 b^2 d x+9 a b^4 d x+ 5 b^3 \left(a^2-b^2\right) \operatorname{Csch}[c+d x]^2-36 a^2 b \left(a^2+b^2\right) \operatorname{Log}[b \operatorname{Cosh}[c+d x]+a \operatorname{Sinh}[c+d x]]\right)+ a^2 \left(18 a^4 b-14 a^2 b^3-4 b^5+3 a^5 (c+d x)+18 a^3 b^2 (c+d x)+3 a b^4 (c+d x)-12 a^2 b \left(a^2+b^2\right) \operatorname{Log}[b \operatorname{Cosh}[c+d x]+a \operatorname{Sinh}[c+d x]]\right)\Big)$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b \operatorname{Coth}[c+d x]}\, dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Coth}[c+d x]}}{\sqrt{a-b}}\right]}{d}+\frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Coth}[c+d x]}}{\sqrt{a+b}}\right]}{d}$$

Result (type 3, 128 leaves):

$$\frac{\left(-\sqrt{i (a-b)} \operatorname{ArcTanh}\left[\frac{\sqrt{i (a+b \operatorname{Coth}[c+d x])}}{\sqrt{i (a-b)}}\right]+\sqrt{i (a+b)} \operatorname{ArcTanh}\left[\frac{\sqrt{i (a+b \operatorname{Coth}[c+d x])}}{\sqrt{i (a+b)}}\right]\right) \sqrt{a+b \operatorname{Coth}[c+d x]}}{d \sqrt{i (a+b \operatorname{Coth}[c+d x])}}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b \operatorname{Coth}[c+d x]}}\, dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Coth}[c+d x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}\, d}+\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Coth}[c+d x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b}\, d}$$

Result (type 3, 129 leaves):

$$-\frac{\left(\frac{\text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a-b)}}\right]}{\sqrt{i\,(a-b)}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a+b)}}\right]}{\sqrt{i\,(a+b)}}\right)\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{d\,\sqrt{a+b\,\text{Coth}[c+d\,x]}}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$\text{ArcTanh}[\text{Cosh}[x]]-\text{Csch}[x]$$

Result (type 3, 21 leaves):

$$-\text{Csch}[x]+\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]$$

Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{1+\text{Coth}[x]}\,\text{Sech}[x]^2\,dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\text{ArcTanh}\left[\sqrt{1+\text{Coth}[x]}\right]+\sqrt{1+\text{Coth}[x]}\,\text{Tanh}[x]$$

Result (type 3, 675 leaves):

$$
\begin{aligned}
&\frac{1}{2}\sqrt{1+\text{Coth}[x]}\,\Bigg(\frac{(1-\mathrm{i})\,\text{ArcTan}\Big[\Big(\frac{1}{2}+\frac{\mathrm{i}}{2}\Big)\sqrt{\mathrm{i}\,(1+\text{Coth}[x])}\Big]}{\sqrt{\mathrm{i}\,(1+\text{Coth}[x])}}+\\
&\frac{1}{2\sqrt{\text{Tanh}\big[\frac{x}{2}\big]}}\Bigg((2+2\,\mathrm{i})\,(-1)^{1/4}\,\text{ArcTan}\Big[\Big((2+\mathrm{i})+2\sqrt{-1-\mathrm{i}}\,\Big(1+(-1)^{1/4}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big)\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+2\,(-1)^{1/4}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}-\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big/\\
&\quad\Big((-2-\mathrm{i})-2\sqrt{-1-\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+(1+2\,\mathrm{i})\,\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big]+\\
&(2+2\,\mathrm{i})\,(-1)^{1/4}\,\text{ArcTan}\Big[\Big((2+\mathrm{i})+(-1-\mathrm{i})^{3/2}\Big((1-\mathrm{i})+\sqrt{2}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big)\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+2\,(-1)^{1/4}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}-\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big/\\
&\quad\Big((-2-\mathrm{i})+2\sqrt{-1-\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+(1+2\,\mathrm{i})\,\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big]+\\
&\sqrt{2}\,\text{Log}\Big[(1-\mathrm{i})\Big(1+\sqrt{2}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big)-\frac{2\Big((1+\mathrm{i})+\mathrm{i}\sqrt{2}\sqrt{\text{Tanh}\big[\frac{x}{2}\big]}\Big)\sqrt{-1+\text{Tanh}\big[\frac{x}{2}\big]}}{\sqrt{-1+\mathrm{i}}}+(2+\mathrm{i})\Big(-1+\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big]+\\
&\sqrt{2}\,\text{Log}\Big[(1-\mathrm{i})\Big(1+\sqrt{2}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big)-(-1+\mathrm{i})^{3/2}\Big((1-\mathrm{i})+\sqrt{2}\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big)\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+(2+\mathrm{i})\Big(-1+\text{Tanh}\big[\tfrac{x}{2}\big]\Big)\Big]-\\
&8\,\text{Log}\Big[1+\sqrt{\text{Tanh}\big[\tfrac{x}{2}\big]}\Big]+\sqrt{2}\,\text{Log}\Big[(2+\mathrm{i})+2\sqrt{-1-\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}-\text{Tanh}\big[\tfrac{x}{2}\big]\Big]+4\,\text{Log}\Big[-1+\text{Tanh}\big[\tfrac{x}{2}\big]\Big]-\\
&\sqrt{2}\,\text{Log}\Big[(-2-\mathrm{i})-2\sqrt{-1-\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+\text{Tanh}\big[\tfrac{x}{2}\big]\Big]-\sqrt{2}\,\text{Log}\Big[(-2+\mathrm{i})-2\sqrt{-1+\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+\text{Tanh}\big[\tfrac{x}{2}\big]\Big]-\\
&\sqrt{2}\,\text{Log}\Big[(-2+\mathrm{i})+2\sqrt{-1+\mathrm{i}}\,\sqrt{-1+\text{Tanh}\big[\tfrac{x}{2}\big]}+\text{Tanh}\big[\tfrac{x}{2}\big]\Big]\Bigg)\Big(\text{Cosh}\big[\tfrac{x}{2}\big]-\text{Sinh}\big[\tfrac{x}{2}\big]\Big)\,\text{Sinh}\big[\tfrac{x}{2}\big]+2\,\text{Tanh}[x]\Bigg)
\end{aligned}
$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]\,(1+\text{Coth}[x])^{3/2}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}-\frac{2}{3}(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 90 leaves):

$$-\left(\left(2\,(1+\text{Coth}[x])^{3/2}\left(\text{Cosh}[x]\sqrt{i\,(1+\text{Coth}[x])}-(3-3\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\text{Sinh}[x]+4\sqrt{i\,(1+\text{Coth}[x])}\,\text{Sinh}[x]\right)\right)\Big/\left(3\sqrt{i\,(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])\right)\right)$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]\sqrt{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 53 leaves):

$$(1+i)\sqrt{1+\text{Coth}[x]}\left((-1+i)-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}\right)$$

Problem 134: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{1+\text{Coth}[x]}}\,dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{1}{\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 97 leaves):

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i+i\,\text{Coth}[x]}\right]\text{Csch}[x]\left(\text{Cosh}[x]+\text{Sinh}[x]\right)}{\sqrt{i+i\,\text{Coth}[x]}\sqrt{1+\text{Coth}[x]}}+\frac{\text{Csch}[x]\left(\text{Cosh}[x]+\text{Sinh}[x]\right)\left(\frac{1}{2}-\frac{1}{2}\text{Cosh}[2x]+\frac{1}{2}\text{Sinh}[2x]\right)}{\sqrt{1+\text{Coth}[x]}}$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Coth}[x]}{(1+\text{Coth}[x])^{3/2}}\,dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\sqrt{2}}+\frac{1}{3(1+\text{Coth}[x])^{3/2}}-\frac{1}{2\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 84 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}+\left(\frac{1}{6}-\frac{i}{6}\right)\left(-2+\text{Cosh}[2x]+\text{Cosh}[4x]-\text{Sinh}[2x]-\text{Sinh}[4x]\right)\right)$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int\text{Coth}[x]^2\,(1+\text{Coth}[x])^{3/2}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}-\frac{2}{5}(1+\text{Coth}[x])^{5/2}$$

Result (type 3, 70 leaves):

$$-\frac{1}{5\sqrt{1+\text{Coth}[x]}}2\left(7+2\,\text{Coth}[x]^2+(5+5\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\sqrt{i\,(1+\text{Coth}[x])}+\text{Csch}[x]^2+\text{Coth}[x]\left(9+\text{Csch}[x]^2\right)\right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int\text{Coth}[x]^2\sqrt{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-\frac{2}{3}\,(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 61 leaves):

$$\frac{-2-4\,\text{Coth}[x]-2\,\text{Coth}[x]^2-(3+3\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\sqrt{i\,(1+\text{Coth}[x])}}{3\,\sqrt{1+\text{Coth}[x]}}$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Coth}[x]^2}{\sqrt{1+\text{Coth}[x]}}\,dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{\sqrt{1+\text{Coth}[x]}}-2\,\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 81 leaves):

$$\frac{1}{\sqrt{1+\text{Coth}[x]}}\left(\frac{1}{2}+\frac{i}{2}\right)\text{Csch}[x]\,(\text{Cosh}[x]+\text{Sinh}[x])\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}+\left(\frac{1}{2}-\frac{i}{2}\right)(-5+\text{Cosh}[2\,x]-\text{Sinh}[2\,x])\right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Coth}[x]^2}{(1+\text{Coth}[x])^{3/2}}\,dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\,\sqrt{2}}-\frac{1}{3\,(1+\text{Coth}[x])^{3/2}}+\frac{3}{2\,\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 86 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}-\left(\frac{1}{6}-\frac{i}{6}\right)(-8+7\,\text{Cosh}[2\,x]+\text{Cosh}[4\,x]-7\,\text{Sinh}[2\,x]-\text{Sinh}[4\,x])\right)$$

Problem 151: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{Coth}[a+2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 30 leaves, 4 steps):

$$\frac{x^4}{4}+\frac{1}{2} e^{-2 a} \operatorname{Log}\left[1-e^{2 a} x^4\right]$$

Result (type 3, 64 leaves):

$$\frac{x^4}{4}+\frac{1}{2} \operatorname{Cosh}[2 a] \operatorname{Log}\left[-\operatorname{Cosh}[a]+x^4 \operatorname{Cosh}[a]+\operatorname{Sinh}[a]+x^4 \operatorname{Sinh}[a]\right]-\frac{1}{2} \operatorname{Log}\left[-\operatorname{Cosh}[a]+x^4 \operatorname{Cosh}[a]+\operatorname{Sinh}[a]+x^4 \operatorname{Sinh}[a]\right] \operatorname{Sinh}[2 a]$$

Problem 152: Result is not expressed in closed-form.

$$\int x^2 \operatorname{Coth}[a+2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{x^3}{3}+e^{-3 a/2} \operatorname{ArcTan}\left[e^{a/2} x\right]-e^{-3 a/2} \operatorname{ArcTanh}\left[e^{a/2} x\right]$$

Result (type 7, 64 leaves):

$$\frac{1}{6}\left(2 x^3+3 \operatorname{RootSum}\left[-\operatorname{Cosh}[a]+\operatorname{Sinh}[a]+\operatorname{Cosh}[a] \#1^4+\operatorname{Sinh}[a] \#1^4 \&, \frac{\operatorname{Log}[x]-\operatorname{Log}[x-\#1]}{\#1} \&\right]\left(-\operatorname{Cosh}[2 a]+\operatorname{Sinh}[2 a]\right)\right)$$

Problem 154: Result is not expressed in closed-form.

$$\int \operatorname{Coth}[a+2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$x-e^{-a/2} \operatorname{ArcTan}\left[e^{a/2} x\right]-e^{-a/2} \operatorname{ArcTanh}\left[e^{a/2} x\right]$$

Result (type 7, 58 leaves):

$$x+\frac{1}{2} \operatorname{RootSum}\left[-\operatorname{Cosh}[a]+\operatorname{Sinh}[a]+\operatorname{Cosh}[a] \#1^4+\operatorname{Sinh}[a] \#1^4 \&, \frac{\operatorname{Log}[x]-\operatorname{Log}[x-\#1]}{\#1^3} \&\right]\left(-\operatorname{Cosh}[2 a]+\operatorname{Sinh}[2 a]\right)$$

Problem 156: Result is not expressed in closed-form.

$$\int \frac{\text{Coth}[a + 2\,\text{Log}[x]]}{x^2}\, dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{1}{x} + e^{a/2}\,\text{ArcTan}\left[e^{a/2}\,x\right] - e^{a/2}\,\text{ArcTanh}\left[e^{a/2}\,x\right]$$

Result (type 7, 62 leaves):

$$\frac{2 + x\,\text{RootSum}\left[-\text{Cosh}[a] - \text{Sinh}[a] + \text{Cosh}[a]\,\#1^4 - \text{Sinh}[a]\,\#1^4\,\&,\ \frac{\text{Log}[x] + \text{Log}\left[\frac{1}{x} - \#1\right]}{\#1^3}\,\&\right]\,(\text{Cosh}[a] + \text{Sinh}[a])^2}{2\,x}$$

Problem 166: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^m\,\text{Coth}[a + 2\,\text{Log}[x]]^2\, dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{(e\,x)^{1+m}}{e\,(1+m)} + \frac{(e\,x)^{1+m}}{e\,\left(1 - e^{2a}\,x^4\right)} - \frac{(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, e^{2a}\,x^4\right]}{e}$$

Result (type 5, 165 leaves):

$$\frac{1}{(\text{Cosh}[a] - \text{Sinh}[a])^2}\, x\,(e\,x)^m \left(\frac{1}{(5+m)\,(9+m)} x^4\,(\text{Cosh}[a] + \text{Sinh}[a])\,\left(2\,(9+m)\,\text{Hypergeometric2F1}\left[2, \frac{5+m}{4}, \frac{9+m}{4}, x^4\,(\text{Cosh}[2a] + \text{Sinh}[2a])\right]\,(\text{Cosh}[a] - \text{Sinh}[a]) + (5+m)\,x^4\,\text{Hypergeometric2F1}\left[2, \frac{9+m}{4}, \frac{13+m}{4}, x^4\,(\text{Cosh}[2a] + \text{Sinh}[2a])\right]\,(\text{Cosh}[a] + \text{Sinh}[a])\right) + \frac{\text{Hypergeometric2F1}\left[2, \frac{1+m}{4}, \frac{5+m}{4}, x^4\,(\text{Cosh}[2a] + \text{Sinh}[2a])\right]\,(\text{Cosh}[2a] - \text{Sinh}[2a])}{1+m}\right)$$

Problem 168: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a + b\,\text{Log}[x]]^p\, dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\text{AppellF1}\left[\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right]$$

Result (type 6, 259 leaves):

$$\left((1+2b)\,x\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p\text{AppellF1}\left[\frac{1}{2b},p,-p,1+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]\right)\Big/\left(2b\,e^{2a}p\,x^{2b}\,\text{AppellF1}\left[1+\frac{1}{2b},p,1-p,2+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]+2b\,e^{2a}p\,x^{2b}\,\text{AppellF1}\left[1+\frac{1}{2b},1+p,-p,2+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]+(1+2b)\,\text{AppellF1}\left[\frac{1}{2b},p,-p,1+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]\right)$$

Problem 169: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^m\,\text{Coth}[a+b\,\text{Log}[x]]^p\,dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{(e\,x)^{1+m}\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\text{AppellF1}\left[\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]}{e\,(1+m)}$$

Result (type 6, 287 leaves):

$$\left((1+2b+m)\,x\,(e\,x)^m\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p\text{AppellF1}\left[\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]\right)\Big/\left((1+m)\left((1+2b+m)\,\text{AppellF1}\left[\frac{1+m}{2b},p,-p,\frac{1+2b+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]+2b\,e^{2a}p\,x^{2b}\left(\text{AppellF1}\left[\frac{1+2b+m}{2b},p,1-p,\frac{1+4b+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]+\text{AppellF1}\left[\frac{1+2b+m}{2b},1+p,-p,\frac{1+4b+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right]\right)\right)\right)$$

Problem 171: Result unnecessarily involves higher level functions.

$$\int \text{Coth}\left[a+\frac{\text{Log}[x]}{4}\right]^p\,dx$$

Optimal (type 5, 108 leaves, 4 steps):

$$e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{2^{1-p}e^{-4a}p\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left[p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt{x}\right)\right]}{1+p}$$

Result (type 6, 176 leaves):

$$\left(3\left(\frac{1+e^{2a}\sqrt{x}}{-1+e^{2a}\sqrt{x}}\right)^p x\,\text{AppellF1}\left[2,p,-p,3,e^{2a}\sqrt{x},-e^{2a}\sqrt{x}\right]\right)\Big/\left(3\,\text{AppellF1}\left[2,p,-p,3,e^{2a}\sqrt{x},-e^{2a}\sqrt{x}\right]+e^{2a}p\sqrt{x}\left(\text{AppellF1}\left[3,p,1-p,4,e^{2a}\sqrt{x},-e^{2a}\sqrt{x}\right]+\text{AppellF1}\left[3,1+p,-p,4,e^{2a}\sqrt{x},-e^{2a}\sqrt{x}\right]\right)\right)$$

Problem 172: Result unnecessarily involves higher level functions.

$$\int \text{Coth}\left[a+\frac{\text{Log}[x]}{6}\right]^p dx$$

Optimal (type 5, 162 leaves, 5 steps):

$$e^{-6a}p\left(-1-e^{2a}x^{1/3}\right)^{1+p}\left(1-e^{2a}x^{1/3}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}x^{1/3}\right)^{1+p}\left(1-e^{2a}x^{1/3}\right)^{1-p}x^{1/3}-\frac{2^{-p}e^{-6a}\left(1+2p^2\right)\left(-1-e^{2a}x^{1/3}\right)^{1+p}\text{Hypergeometric2F1}\left[p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}x^{1/3}\right)\right]}{1+p}$$

Result (type 6, 176 leaves):

$$\left(4\left(\frac{1+e^{2a}x^{1/3}}{-1+e^{2a}x^{1/3}}\right)^p x\,\text{AppellF1}\left[3,p,-p,4,e^{2a}x^{1/3},-e^{2a}x^{1/3}\right]\right)\Big/\left(4\,\text{AppellF1}\left[3,p,-p,4,e^{2a}x^{1/3},-e^{2a}x^{1/3}\right]+e^{2a}p\,x^{1/3}\left(\text{AppellF1}\left[4,p,1-p,5,e^{2a}x^{1/3},-e^{2a}x^{1/3}\right]+\text{AppellF1}\left[4,1+p,-p,5,e^{2a}x^{1/3},-e^{2a}x^{1/3}\right]\right)\right)$$

Problem 173: Result unnecessarily involves higher level functions.

$$\int \text{Coth}\left[a+\frac{\text{Log}[x]}{8}\right]^p dx$$

Optimal (type 5, 194 leaves, 5 steps):

$$\frac{1}{3}e^{-12a}\left(-1-e^{2a}x^{1/4}\right)^{1+p}\left(1-e^{2a}x^{1/4}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)+2e^{6a}p\,x^{1/4}\right)+e^{-4a}\left(-1-e^{2a}x^{1/4}\right)^{1+p}\left(1-e^{2a}x^{1/4}\right)^{1-p}\sqrt{x}-\frac{2^{2-p}e^{-8a}p\left(2+p^2\right)\left(-1-e^{2a}x^{1/4}\right)^{1+p}\text{Hypergeometric2F1}\left[p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}x^{1/4}\right)\right]}{3(1+p)}$$

Result (type 6, 176 leaves):

$$\left(5\left(\frac{1+e^{2a}x^{1/4}}{-1+e^{2a}x^{1/4}}\right)^p x\,\text{AppellF1}\left[4,p,-p,5,e^{2a}x^{1/4},-e^{2a}x^{1/4}\right]\right)\Big/\left(5\,\text{AppellF1}\left[4,p,-p,5,e^{2a}x^{1/4},-e^{2a}x^{1/4}\right]+e^{2a}p\,x^{1/4}\left(\text{AppellF1}\left[5,p,1-p,6,e^{2a}x^{1/4},-e^{2a}x^{1/4}\right]+\text{AppellF1}\left[5,1+p,-p,6,e^{2a}x^{1/4},-e^{2a}x^{1/4}\right]\right)\right)$$

Problem 174: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a+\text{Log}[x]]^p \, dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x\left(-1-e^{2a}x^2\right)^p\left(1+e^{2a}x^2\right)^{-p}\text{AppellF1}\left[\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right]$$

Result (type 6, 171 leaves):

$$\left(3x\left(\frac{1+e^{2a}x^2}{-1+e^{2a}x^2}\right)^p\text{AppellF1}\left[\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right]\right)\Big/\left(3\,\text{AppellF1}\left[\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right]+2e^{2a}p\,x^2\left(\text{AppellF1}\left[\frac{3}{2},p,1-p,\frac{5}{2},e^{2a}x^2,-e^{2a}x^2\right]+\text{AppellF1}\left[\frac{3}{2},1+p,-p,\frac{5}{2},e^{2a}x^2,-e^{2a}x^2\right]\right)\right)$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a+2\,\text{Log}[x]]^p \, dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x\left(-1-e^{2a}x^4\right)^p\left(1+e^{2a}x^4\right)^{-p}\text{AppellF1}\left[\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right]$$

Result (type 6, 171 leaves):

$$\left(5x\left(\frac{1+e^{2a}x^4}{-1+e^{2a}x^4}\right)^p\text{AppellF1}\left[\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right]\right)\Big/\left(5\,\text{AppellF1}\left[\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right]+4e^{2a}p\,x^4\left(\text{AppellF1}\left[\frac{5}{4},p,1-p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right]+\text{AppellF1}\left[\frac{5}{4},1+p,-p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right]\right)\right)$$

Problem 176: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a+3\,\text{Log}[x]]^p \, dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x\left(-1-e^{2a}x^6\right)^p\left(1+e^{2a}x^6\right)^{-p}\text{AppellF1}\left[\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right]$$

Result (type 6, 171 leaves):

$$\left(7\,x\left(\frac{1+e^{2a}x^6}{-1+e^{2a}x^6}\right)^p \text{AppellF1}\left[\frac{1}{6},\,p,\,-p,\,\frac{7}{6},\,e^{2a}x^6,\,-e^{2a}x^6\right]\right)\Big/\left(7\,\text{AppellF1}\left[\frac{1}{6},\,p,\,-p,\,\frac{7}{6},\,e^{2a}x^6,\,-e^{2a}x^6\right]+6\,e^{2a}\,p\,x^6\left(\text{AppellF1}\left[\frac{7}{6},\,p,\,1-p,\,\frac{13}{6},\,e^{2a}x^6,\,-e^{2a}x^6\right]+\text{AppellF1}\left[\frac{7}{6},\,1+p,\,-p,\,\frac{13}{6},\,e^{2a}x^6,\,-e^{2a}x^6\right]\right)\right)$$

Problem 177: Result more than twice size of optimal antiderivative.

$$\int x^3\,\text{Coth}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,dx$$

Optimal (type 5, 58 leaves, 4 steps):

$$\frac{x^4}{4}-\frac{1}{2}x^4\,\text{Hypergeometric2F1}\left[1,\,\frac{2}{b\,d\,n},\,1+\frac{2}{b\,d\,n},\,e^{2ad}\left(c\,x^n\right)^{2bd}\right]$$

Result (type 5, 198 leaves):

$$-\frac{1}{8+4\,b\,d\,n}x^4\left(2\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\,\text{Hypergeometric2F1}\left[1,\,1+\frac{2}{b\,d\,n},\,2+\frac{2}{b\,d\,n},\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\right]+(2+b\,d\,n)\left(\text{Coth}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\text{Coth}\left[d\,\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]+\text{Hypergeometric2F1}\left[1,\,\frac{2}{b\,d\,n},\,1+\frac{2}{b\,d\,n},\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\right]+\text{Csch}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Csch}\left[d\,\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Sinh}\left[b\,d\,n\,\text{Log}[x]\right]\right)\right)$$

Problem 178: Result more than twice size of optimal antiderivative.

$$\int x^2\,\text{Coth}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,dx$$

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{x^3}{3}-\frac{2}{3}x^3\,\text{Hypergeometric2F1}\left[1,\,\frac{3}{2\,b\,d\,n},\,1+\frac{3}{2\,b\,d\,n},\,e^{2ad}\left(c\,x^n\right)^{2bd}\right]$$

Result (type 5, 207 leaves):

$$-\frac{1}{9+6\,b\,d\,n}x^3\left(3\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\,\text{Hypergeometric2F1}\left[1,\,1+\frac{3}{2\,b\,d\,n},\,2+\frac{3}{2\,b\,d\,n},\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\right]+(3+2\,b\,d\,n)\left(\text{Coth}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\text{Coth}\left[d\,\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]+\text{Hypergeometric2F1}\left[1,\,\frac{3}{2\,b\,d\,n},\,1+\frac{3}{2\,b\,d\,n},\,e^{2d\,(a+b\,\text{Log}[c\,x^n])}\right]+\text{Csch}\left[d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Csch}\left[d\,\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Sinh}\left[b\,d\,n\,\text{Log}[x]\right]\right)\right)$$

Problem 179: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Coth}[d (a + b \operatorname{Log}[c x^n])] \, dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{x^2}{2} - x^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 5, 193 leaves):

$$-\frac{1}{2 + 2 b d n} x^2 \left(e^{2 d (a+b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{b d n}, 2 + \frac{1}{b d n}, e^{2 d (a+b \operatorname{Log}[c x^n])}\right] + (1 + b d n) \left(\operatorname{Coth}[d (a + b \operatorname{Log}[c x^n])] - \operatorname{Coth}[d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] + \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, e^{2 d (a+b \operatorname{Log}[c x^n])}\right] + \operatorname{Csch}[d (a + b \operatorname{Log}[c x^n])] \operatorname{Csch}[d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] \operatorname{Sinh}[b d n \operatorname{Log}[x]]\right)\right)$$

Problem 180: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[d (a + b \operatorname{Log}[c x^n])] \, dx$$

Optimal (type 5, 52 leaves, 4 steps):

$$x - 2 x \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 5, 199 leaves):

$$-\frac{e^{2 a d} x (c x^n)^{2 b d} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2 b d n}, 2 + \frac{1}{2 b d n}, e^{2 d (a+b \operatorname{Log}[c x^n])}\right]}{1 + 2 b d n} - x \left(\operatorname{Coth}[d (a + b \operatorname{Log}[c x^n])] - \operatorname{Coth}[d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] + \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, e^{2 d (a+b \operatorname{Log}[c x^n])}\right] + \operatorname{Csch}[d (a + b \operatorname{Log}[c x^n])] \operatorname{Csch}[d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] \operatorname{Sinh}[b d n \operatorname{Log}[x]]\right)$$

Problem 182: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[d (a + b \operatorname{Log}[c x^n])]}{x^2} \, dx$$

Optimal (type 5, 58 leaves, 4 steps):

$$-\frac{1}{x}+\frac{2\,\text{Hypergeometric2F1}\left[1,-\frac{1}{2\,b\,d\,n},1-\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\left(c\,x^n\right)^{2\,b\,d}\right]}{x}$$

Result (type 5, 197 leaves):

$$\frac{1}{x}\left(\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\text{Coth}\left[d\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\frac{e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\,\text{Hypergeometric2F1}\left[1,1-\frac{1}{2\,b\,d\,n},2-\frac{1}{2\,b\,d\,n},e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\right]}{-1+2\,b\,d\,n}+\text{Hypergeometric2F1}\left[1,-\frac{1}{2\,b\,d\,n},1-\frac{1}{2\,b\,d\,n},e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\right]+\text{Csch}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Csch}\left[d\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Sinh}\left[b\,d\,n\,\text{Log}[x]\right]\right)$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]}{x^3}\,dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{1}{2\,x^2}+\frac{\text{Hypergeometric2F1}\left[1,-\frac{1}{b\,d\,n},1-\frac{1}{b\,d\,n},e^{2\,a\,d}\left(c\,x^n\right)^{2\,b\,d}\right]}{x^2}$$

Result (type 5, 191 leaves):

$$\frac{1}{2\,x^2}\left(\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\text{Coth}\left[d\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]-\frac{e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\,\text{Hypergeometric2F1}\left[1,1-\frac{1}{b\,d\,n},2-\frac{1}{b\,d\,n},e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\right]}{-1+b\,d\,n}+\text{Hypergeometric2F1}\left[1,-\frac{1}{b\,d\,n},1-\frac{1}{b\,d\,n},e^{2\,d\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}\right]+\text{Csch}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Csch}\left[d\left(a-b\,n\,\text{Log}[x]+b\,\text{Log}\left[c\,x^n\right]\right)\right]\,\text{Sinh}\left[b\,d\,n\,\text{Log}[x]\right]\right)$$

Problem 196: Attempted integration timed out after 120 seconds.

$$\int(e\,x)^m\,\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^n\right]\right)\right]^3\,dx$$

Optimal (type 5, 306 leaves, 6 steps):

$$\frac{(1+m+b\,d\,n)\ (1+m+2\,b\,d\,n)\ (e\,x)^{1+m}}{2\,b^2\,d^2\,e\,(1+m)\,n^2}-\frac{(e\,x)^{1+m}\left(1+e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^2}{2\,b\,d\,e\,n\left(1-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^2}+\frac{e^{-2\,a\,d}\,(e\,x)^{1+m}\left(\frac{e^{2\,a\,d}\,(1+m-2\,b\,d\,n)}{n}+\frac{e^{4\,a\,d}\,(1+m+2\,b\,d\,n)\,(c\,x^n)^{2\,b\,d}}{n}\right)}{2\,b^2\,d^2\,e\,n\left(1-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)}-$$

$$\frac{(1+2\,m+m^2+2\,b^2\,d^2\,n^2)\ (e\,x)^{1+m}\ \text{Hypergeometric2F1}\left[1,\frac{1+m}{2\,b\,d\,n},1+\frac{1+m}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]}{b^2\,d^2\,e\,(1+m)\,n^2}$$

Result (type 1, 1 leaves):

???

Problem 197: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[d\,(a+b\,\text{Log}[c\,x^n])]^p\,dx$$

Optimal (type 6, 115 leaves, 4 steps):

$$x\left(-1-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^p\left(1+e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^{-p}\text{AppellF1}\left[\frac{1}{2\,b\,d\,n},p,-p,1+\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]$$

Result (type 6, 387 leaves):

$$\left((1+2\,b\,d\,n)\ x\left(\frac{1+e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}}{-1+e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}}\right)^p\text{AppellF1}\left[\frac{1}{2\,b\,d\,n},p,-p,1+\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]\right)\Big/$$

$$\Big(2\,b\,d\,e^{2\,a\,d}\,n\,p\,(c\,x^n)^{2\,b\,d}\,\text{AppellF1}\left[1+\frac{1}{2\,b\,d\,n},p,1-p,2+\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]+$$

$$2\,b\,d\,e^{2\,a\,d}\,n\,p\,(c\,x^n)^{2\,b\,d}\,\text{AppellF1}\left[1+\frac{1}{2\,b\,d\,n},1+p,-p,2+\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]+$$

$$(1+2\,b\,d\,n)\ \text{AppellF1}\left[\frac{1}{2\,b\,d\,n},p,-p,1+\frac{1}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]\Big)$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^m\,\text{Coth}[d\,(a+b\,\text{Log}[c\,x^n])]^p\,dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e\,(1+m)}(e\,x)^{1+m}\left(-1-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^p\left(1+e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right)^{-p}\text{AppellF1}\left[\frac{1+m}{2\,b\,d\,n},p,-p,1+\frac{1+m}{2\,b\,d\,n},e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d},-e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}\right]$$

Result (type 6, 417 leaves):

$$\left((1+m+2bdn)\,x\,(e x)^m\left(\frac{1+e^{2ad}(c x^n)^{2bd}}{-1+e^{2ad}(c x^n)^{2bd}}\right)^p \text{AppellF1}\left[\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(c x^n)^{2bd},-e^{2ad}(c x^n)^{2bd}\right]\right)\Big/$$
$$\left((1+m)\left((1+m+2bdn)\,\text{AppellF1}\left[\frac{1+m}{2bdn},p,-p,\frac{1+m+2bdn}{2bdn},e^{2ad}(c x^n)^{2bd},-e^{2ad}(c x^n)^{2bd}\right]+\right.\right.$$
$$2bd\,e^{2ad}\,n\,p\,(c x^n)^{2bd}\left(\text{AppellF1}\left[\frac{1+m+2bdn}{2bdn},p,1-p,\frac{1+m+4bdn}{2bdn},e^{2ad}(c x^n)^{2bd},-e^{2ad}(c x^n)^{2bd}\right]+\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{1+m+2bdn}{2bdn},1+p,-p,\frac{1+m+4bdn}{2bdn},e^{2ad}(c x^n)^{2bd},-e^{2ad}(c x^n)^{2bd}\right]\right)\right)\right)$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^5}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\frac{(b-2c)\,\text{ArcTanh}\left[\frac{b+2c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4c^{3/2}}+\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}{2c}$$

Result (type 3, 42 946 leaves): Display of huge result suppressed!

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{b+2c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{c}}+\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 3, 27 092 leaves): Display of huge result suppressed!

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 58 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 3, 27 092 leaves): Display of huge result suppressed!

Problem 208: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a+b\,\text{Coth}[x]^2}{2\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a}}+\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 1, 1 leaves):

???

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 183 leaves, 11 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a+b\,\text{Coth}[x]^2}{2\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a}}+\frac{b\,\text{ArcTanh}\left[\frac{2a+b\,\text{Coth}[x]^2}{2\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4a^{3/2}}+$$
$$\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}\;\text{Tanh}[x]^2}{2a}$$

Result (type 3, 42 369 leaves): Display of huge result suppressed!

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x] \sqrt{a + b\,\text{Coth}[x]^2 + c\,\text{Coth}[x]^4}\, dx$$

Optimal (type 3, 132 leaves, 8 steps):

$$-\frac{(b + 2c)\,\text{ArcTanh}\left[\frac{b+2\,c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\,\text{ArcTanh}\left[\frac{2a + b + (b + 2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a + b\,\text{Coth}[x]^2 + c\,\text{Coth}[x]^4}}\right] - \frac{1}{2}\sqrt{a + b\,\text{Coth}[x]^2 + c\,\text{Coth}[x]^4}$$

Result (type 3, 81 208 leaves): Display of huge result suppressed!

Test results for the 53 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 - \text{Coth}[x]^2}\, dx$$

Optimal (type 3, 3 leaves, 3 steps):

$$\text{ArcSin}[\text{Coth}[x]]$$

Result (type 3, 30 leaves):

$$\sqrt{-\text{Csch}[x]^2}\,\left(-\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\,\text{Sinh}[x]$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \left(1 - \text{Coth}[x]^2\right)^{3/2} dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{1}{2}\text{ArcSin}[\text{Coth}[x]] + \frac{1}{2}\text{Coth}[x]\,\sqrt{-\text{Csch}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8}\sqrt{-\text{Csch}[x]^2}\,\left(\text{Csch}\left[\frac{x}{2}\right]^2 - 4\,\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] + 4\,\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] + \text{Sech}\left[\frac{x}{2}\right]^2\right)\,\text{Sinh}[x]$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]^2 \sqrt{a+b\,\text{Coth}[x]^2}\,dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{(a+2b)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{2\sqrt{b}}+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-\frac{1}{2}\text{Coth}[x]\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 3, 191 leaves):

$$-\left(\left(\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\left(\sqrt{2}\sqrt{a+b}\,(a+2b)\,\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]+\sqrt{b}\left(-2\sqrt{2}\,(a+b)\,\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]+\sqrt{a+b}\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Coth}[x]\,\text{Csch}[x]\right)\right)\text{Sinh}[x]\right)\Big/\left(2\sqrt{2}\sqrt{b}\sqrt{a+b}\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]\sqrt{a+b\,\text{Coth}[x]^2}\,dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]-\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Csch}[x]-\frac{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}{\sqrt{2}}}{\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b\,\text{Coth}[x]^2}\,dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]$$

Result (type 3, 137 leaves):

$$\frac{1}{2}\left(-\sqrt{a+b}\,\text{Log}[1-\text{Coth}[x]]+\sqrt{a+b}\,\text{Log}[1+\text{Coth}[x]]-2\sqrt{b}\,\text{Log}\left[b\,\text{Coth}[x]+\sqrt{b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]-\sqrt{a+b}\,\text{Log}\left[a-b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]+\sqrt{a+b}\,\text{Log}\left[a+b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]\,dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]$$

Result (type 3, 134 leaves):

$$\left(\left(\sqrt{-a}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{-a}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}+\sqrt{b}\,\sqrt{a+b}\,\text{ArcSinh}\left[\frac{\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{b}}\right]\sqrt{\frac{-a+b+(a+b)\,\text{Cosh}[2x]}{b}}\right)\text{Csch}[x]\right)\Big/\left(\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\right)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]^2\,dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$\sqrt{a+b}\ \text{ArcTanh}\left[\frac{\sqrt{a+b}\ \text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-\sqrt{a+b\,\text{Coth}[x]^2}\ \text{Tanh}[x]$$

Result (type 3, 114 leaves):

$$\left(\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\left(2\sqrt{a+b}\ \text{ArcTanh}\left[\frac{\sqrt{2}\ \sqrt{a+b}\ \text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Sinh}[x]-\sqrt{2}\ \sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\ \text{Tanh}[x]\right)\right)\Big/\left(2\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 26: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(a+b\,\text{Coth}[x]^2\right)^{3/2}\text{Tanh}[x]\,dx$$

Optimal (type 3, 71 leaves, 8 steps):

$$-a^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]+(a+b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]-b\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 4, 1088 leaves):

$$-b\sqrt{\frac{-a+b+a\,\text{Cosh}[2x]+b\,\text{Cosh}[2x]}{-1+\text{Cosh}[2x]}}+$$

$$\frac{1}{2}\left(-\left(\left(\left(i\left(-3a^2+2ab+b^2\right)(1+\text{Cosh}[x])\sqrt{\frac{-1+\text{Cosh}[2x]}{(1+\text{Cosh}[x])^2}}\sqrt{\frac{-a+b+(a+b)\,\text{Cosh}[2x]}{-1+\text{Cosh}[2x]}}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\ \text{Tanh}\left[\frac{x}{2}\right]\right],\frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\right]-2\,\text{EllipticPi}\left[\frac{2a+b+2\sqrt{a(a+b)}}{b},i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\ \text{Tanh}\left[\frac{x}{2}\right]\right],\frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\right]\right)\text{Tanh}\left[\frac{x}{2}\right]\sqrt{\frac{2a+b+2\sqrt{a(a+b)}+b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b+2\sqrt{a(a+b)}}}\sqrt{1+\frac{b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b-2\sqrt{a(a+b)}}}\right)\Big/\right.\right.\right.$$

$$\left(\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\sqrt{-a+b+(a+b)\text{Cosh}[2x]}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)\sqrt{\frac{4a\,\text{Tanh}\left[\frac{x}{2}\right]^2+b\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}}\right)\Bigg)+$$

$$\frac{1}{\sqrt{-a+b+(a+b)\text{Cosh}[2x]}}3\left(a^2+2ab+b^2\right)\sqrt{-1+\text{Cosh}[2x]}\sqrt{\frac{-a+b+(a+b)\text{Cosh}[2x]}{-1+\text{Cosh}[2x]}}$$

$$\Bigg(-\Bigg(\Bigg(\mathbb{i}\left(1+\text{Cosh}[x]\right)\sqrt{\frac{-1+\text{Cosh}[2x]}{(1+\text{Cosh}[x])^2}}\Bigg(\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\text{Tanh}\left[\frac{x}{2}\right]\right],\frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\right]-2$$

$$\text{EllipticPi}\left[\frac{2a+b+2\sqrt{a(a+b)}}{b},\mathbb{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\text{Tanh}\left[\frac{x}{2}\right]\right],\frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\right]\Bigg)$$

$$\text{Tanh}\left[\frac{x}{2}\right]\sqrt{\frac{2a+b+2\sqrt{a(a+b)}+b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b+2\sqrt{a(a+b)}}}\sqrt{1+\frac{b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b-2\sqrt{a(a+b)}}}\Bigg)\Big/$$

$$\left(\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\sqrt{-1+\text{Cosh}[2x]}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)\sqrt{\frac{4a\,\text{Tanh}\left[\frac{x}{2}\right]^2+b\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}}\right)\Bigg)+$$

$$\Bigg(4\sqrt{2b+a(-1+\text{Cosh}[2x])+b(-1+\text{Cosh}[2x])}\left(-\frac{\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{-1+\text{Cosh}[2x]}}{\sqrt{a(-1+\text{Cosh}[2x])+b(1+\text{Cosh}[2x])}}\right]}{\sqrt{a}}+\frac{1}{\sqrt{a+b}}\right.$$

$$\left.\text{Log}\left[a\sqrt{-1+\text{Cosh}[2x]}+b\sqrt{-1+\text{Cosh}[2x]}+\sqrt{a+b}\sqrt{a(-1+\text{Cosh}[2x])+b(1+\text{Cosh}[2x])}\right]\right)$$

$$\left.\left.\left.\text{Sinh}[x]^2\,\text{Sinh}[2x]\,\text{Tanh}[x]\right)\middle/\left(3\,(-1+\text{Cosh}[2x])^2\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\right)\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\,\text{Coth}[x]^2\right)^{3/2}\text{Tanh}[x]^2\,dx$$

Optimal (type 3, 77 leaves, 7 steps):

$$-b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]+(a+b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-a\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]$$

Result (type 3, 180 leaves):

$$\left(\left(-\sqrt{2}\,b^{3/2}\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]+\sqrt{2}\,(a+b)^2\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]-\right.\right.$$
$$\left.\left.a\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\,\text{Tanh}[x]\right)\middle/\left(\sqrt{2}\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int\left(1+\text{Coth}[x]^2\right)^{3/2}dx$$

Optimal (type 3, 50 leaves, 6 steps):

$$-\frac{5}{2}\,\text{ArcSinh}[\text{Coth}[x]]+2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Coth}[x]}{\sqrt{1+\text{Coth}[x]^2}}\right]-\frac{1}{2}\,\text{Coth}[x]\sqrt{1+\text{Coth}[x]^2}$$

Result (type 3, 116 leaves):

$$-\frac{1}{8}\left(1+\text{Coth}[x]^2\right)^{3/2}\text{Sech}[2x]^2\left(16\,\text{ArcTanh}\left[\frac{\text{Cosh}[x]}{\sqrt{\text{Cosh}[2x]}}\right]\sqrt{\text{Cosh}[2x]}\,\text{Sinh}[x]^3+\right.$$
$$\left.4\left(\text{ArcTan}\left[\frac{\text{Cosh}[x]}{\sqrt{-\text{Cosh}[2x]}}\right]\sqrt{-\text{Cosh}[2x]}-4\sqrt{2}\,\sqrt{\text{Cosh}[2x]}\,\text{Log}\left[\sqrt{2}\,\text{Cosh}[x]+\sqrt{\text{Cosh}[2x]}\right]\right)\text{Sinh}[x]^3+\text{Sinh}[4x]\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{b}$$

Result (type 3, 98 leaves):

$$\frac{1}{2}\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2\,x]\right)\text{Csch}[x]^2}\left(-\frac{\sqrt{2}}{b}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}}\right]\text{Sinh}[x]}{\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}}\right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}$$

Result (type 3, 134 leaves):

$$\left(\left(-\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}}\right]+\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}}\right]\right)\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2\,x]\right)\text{Csch}[x]^2}\,\text{Sinh}[x]\right)\Big/$$
$$\left(\sqrt{b}\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 82 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2\,x]}\,\,\text{Csch}[x]}{\sqrt{a+b}\,\sqrt{(-a+b+(a+b)\,\text{Cosh}[2\,x])\,\text{Csch}[x]^2}}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}$$

Result (type 3, 83 leaves):

$$\frac{1}{2\sqrt{a+b}}\left(-\text{Log}[1-\text{Coth}[x]]+\text{Log}[1+\text{Coth}[x]]-\text{Log}\left[a-b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]+\text{Log}\left[a+b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]\right)$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 127 leaves):

$$-\frac{\left(\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{-a}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]}{\sqrt{-a}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]}{\sqrt{a+b}}\right)\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Csch}[x]}{\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]}{a}$$

Result (type 3, 126 leaves):

$$\left(\left(\sqrt{2}\,a\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]-\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\,\text{Tanh}[x]\right)\Big/\left(\sqrt{2}\,a\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^2}{\left(a+b\,\text{Coth}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{(a+b)^{3/2}}-\frac{\text{Coth}[x]}{(a+b)\,\sqrt{a+b\,\text{Coth}[x]^2}}$$

Result (type 3, 135 leaves):

$$\left(\left(-2\sqrt{a+b}\,\text{Cosh}[x]\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}+\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\right)\right.$$
$$\left.\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\,\text{Sinh}[x]\right)\Big/\left(\sqrt{2}\,(a+b)^{3/2}\,\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)^{3/2}\right)$$

Problem 51: Unable to integrate problem.

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^4}\,dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{1}{2}\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^4}}\right]+\frac{1}{2}\sqrt{a+b}\,\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]-\frac{1}{2}\sqrt{a+b\,\text{Coth}[x]^4}$$

Result (type 8, 17 leaves):

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^4}\,dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b}}$$

Result (type 8, 17 leaves):

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^4}}\,dx$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\left(a+b\,\text{Coth}[x]^4\right)^{3/2}}\,dx$$

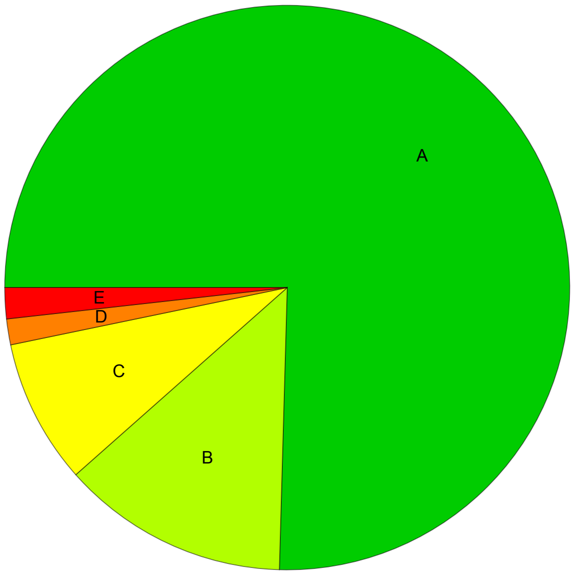
Optimal (type 3, 74 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]}{2\,(a+b)^{3/2}}-\frac{a-b\,\text{Coth}[x]^2}{2\,a\,(a+b)\,\sqrt{a+b\,\text{Coth}[x]^4}}$$

Result (type 3, 31 578 leaves): Display of huge result suppressed!

Summary of Integration Test Results

338 integration problems



A - 255 optimal antiderivatives

B - 44 more than twice size of optimal antiderivatives

C - 28 unnecessarily complex antiderivatives

D - 5 unable to integrate problems

E - 6 integration timeouts